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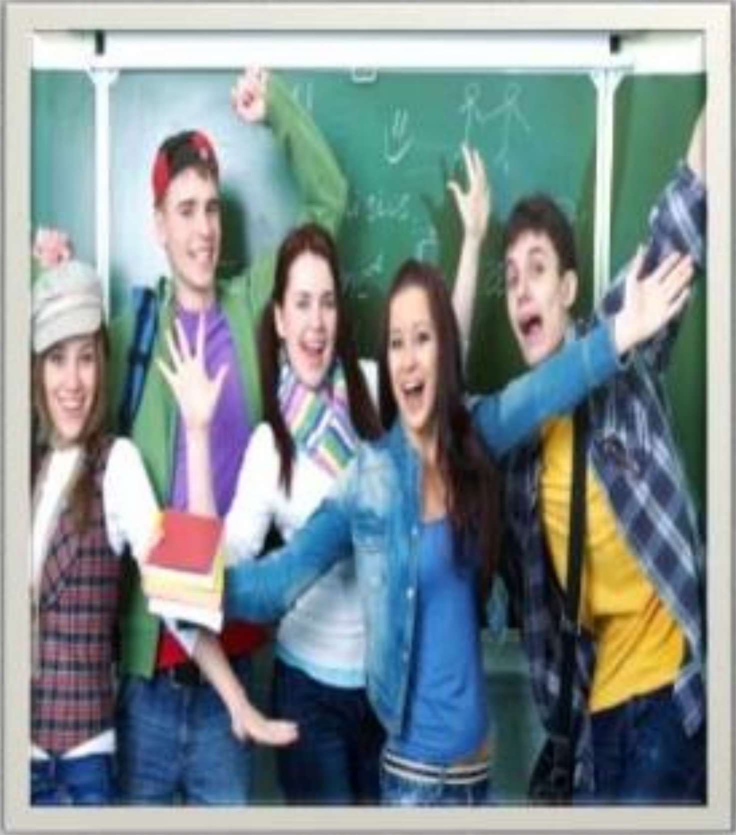
- The term adolescence comes from the latin word ***adolescere***, meaning “to grow to maturity”.
- Primitive people didn’t consider puberty and adolescence to be distinct periods in the life span, the child is regarded as an adult when capable of reproduction
- As it is used today, the term adolescence has a broader meaning. It includes mental, emotional, and social maturity as well as physical maturity.

DEFINITION OF ADOLESCENCE

- Adolescence is a transitional stage of physical and mental development that occurs between childhood and adulthood.
- WHO : period of life between 10 and 19 years.



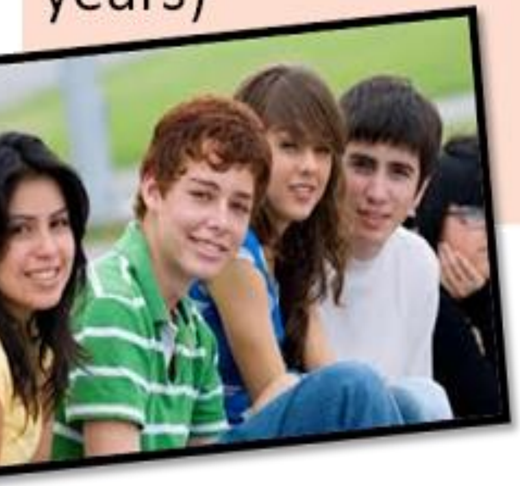
DEFINITION

- Psychologically, adolescence is the age when the individual becomes integrated into the society of adults , the age when the child no longer feels that he is below the level of his elders but equal, at least in rights. This integration into adult society has many affective aspects , more or less linked with puberty. It also includes very profound intellectual changes.

CHARACTERISTICS OF ADOLESCENCE

- ADOLESCENCE IS AN IMPORTANT PERIOD.

Three Phases of Adolescence	What Happens?
Early Adolescence (11-14 years)	A time of rapid pubertal change.
Middle Adolescence (14-16 years)	Pubertal changes are now nearly complete.
Late Adolescence (16-18 years)	The young person achieves full adult appearance and anticipates assumption of adult roles.



- ADOLESCENCE IS A TRANSITIONAL PERIOD.


UNCLE WALLY, WILL YOU PLAY WITH ME ?
WHERE ARE ALL YOUR FRIENDS?
Jan Eliot 2-16
CARTOONSTOCK .com
Search ID: jel110216
© Jan Eliot/Distributed by Universal Uclick via CartoonStock.com
ALL OF A SUDDEN ALL THEY CARE ABOUT IS BOYS... AND MAKE-UP.
EW.
SEE, THAT'S WHY I LOVE YOU. YOU KNOW WHEN TO SAY EW.
WANNA BUILD A FORT?
CARTOONSTOCK .com
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Adolescence

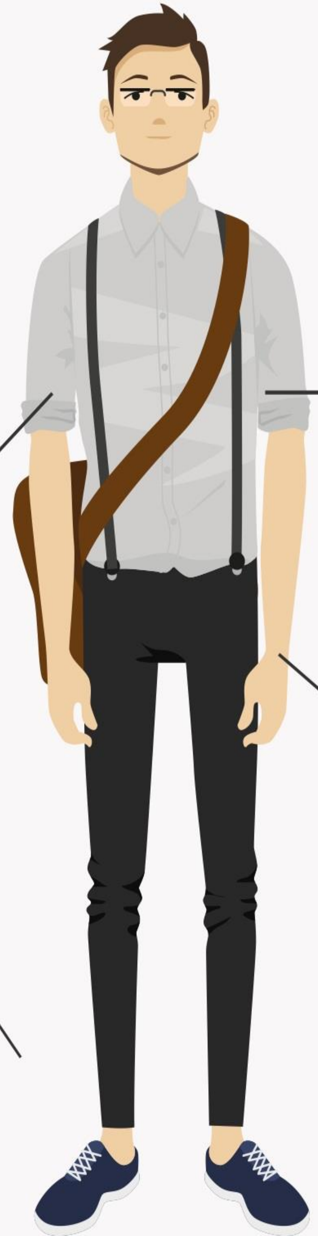
" Physical development is public-everyone sees how tall, short, heavy, thin, muscular, or coordinated you are" (Woolfolk, 68)

Transition from childhood→ adulthood

changes in bodily functions and appearance

Strong natural preoccupation with appearance

self-conscious and self-critical



Intense and variable emotional states

forming own identity
separating themselves from parents

High need for support and acceptance

- ADOLESCENCE IS A PERIOD OF CHANGE.

Key Changes During Adolescence
(Continued)

Adolescents also experience psychological and emotional changes:

■ Mood swings ■ Insecurities, fears, and doubts ■ Behavioural expressions of emotion, which may include withdrawal, hostility, impulsiveness, non-cooperation ■ Self-centeredness ■ Feelings of being misunderstood and/or rejected	■ Fluctuating self-esteem ■ Interest in physical changes, sex, and sexuality ■ Concern about body image ■ Concern about sexual identity, decision-making, and reputation ■ A need to feel autonomous and independent

Typical Physical Changes in Adolescence, p. 259

Table 10-2 Typical Physical Changes in Adolescence

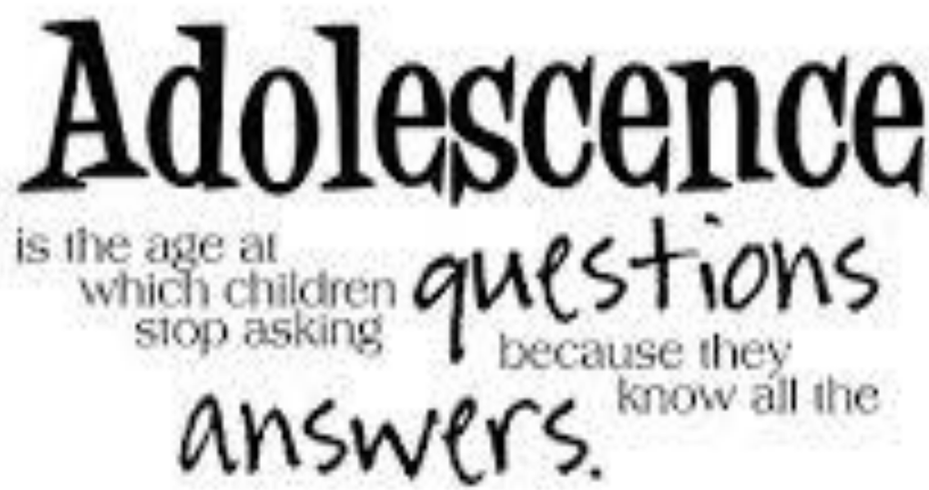
Changes in girls	Changes in boys
• Breast development • Growth of pubic hair • Growth of underarm hair • Body growth • Menarche • Increased output of oil- and sweat-producing glands	• Growth of testes and scrotal sac • Growth of pubic hair • Growth of facial and underarm hair • Body growth • Growth of penis • Change in voice • First ejaculation of semen • Increased output of oil- and sweat-producing glands

- ADOLESCENCE IS A PROBLEM AGE.


STRESS
SPORTS
SCHOOLWORK
MY FUTURE
HORMONES

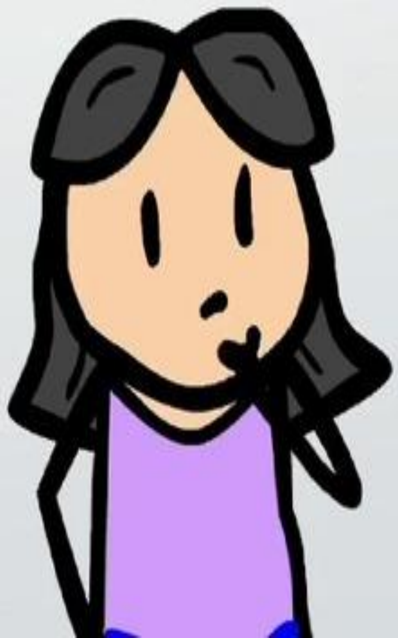
- ADOLESCENCE IS A TIME OF SEARCH FOR IDENTITY.


Who Am I?


Adolescence
is the age at which children stop asking questions because they know all the answers.

Foreclosure

commitments to beliefs and a future, but without truly exploring options



Study.com

- ADOLESCENCE IS DREADED AGE.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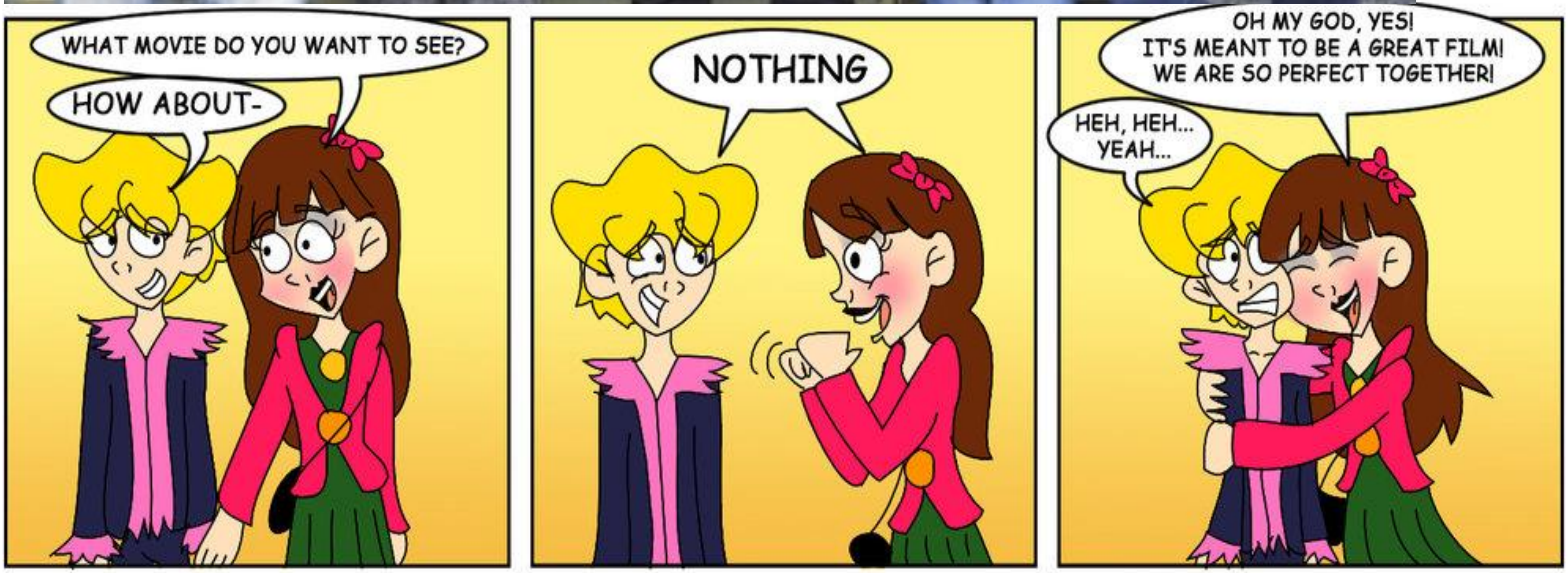
- ADOLESCENCE IS A TIME OF UNREALISM.


WHAT MOVIE DO YOU WANT TO SEE?
HOW ABOUT-
NOTHING
OH MY GOD, YES! IT'S MEANT TO BE A GREAT FILM! WE ARE SO PERFECT TOGETHER!
HEH, HEH... YEAH...

- ADOLESCENCE IS THE THRESHOLD OF ADULTHOOD.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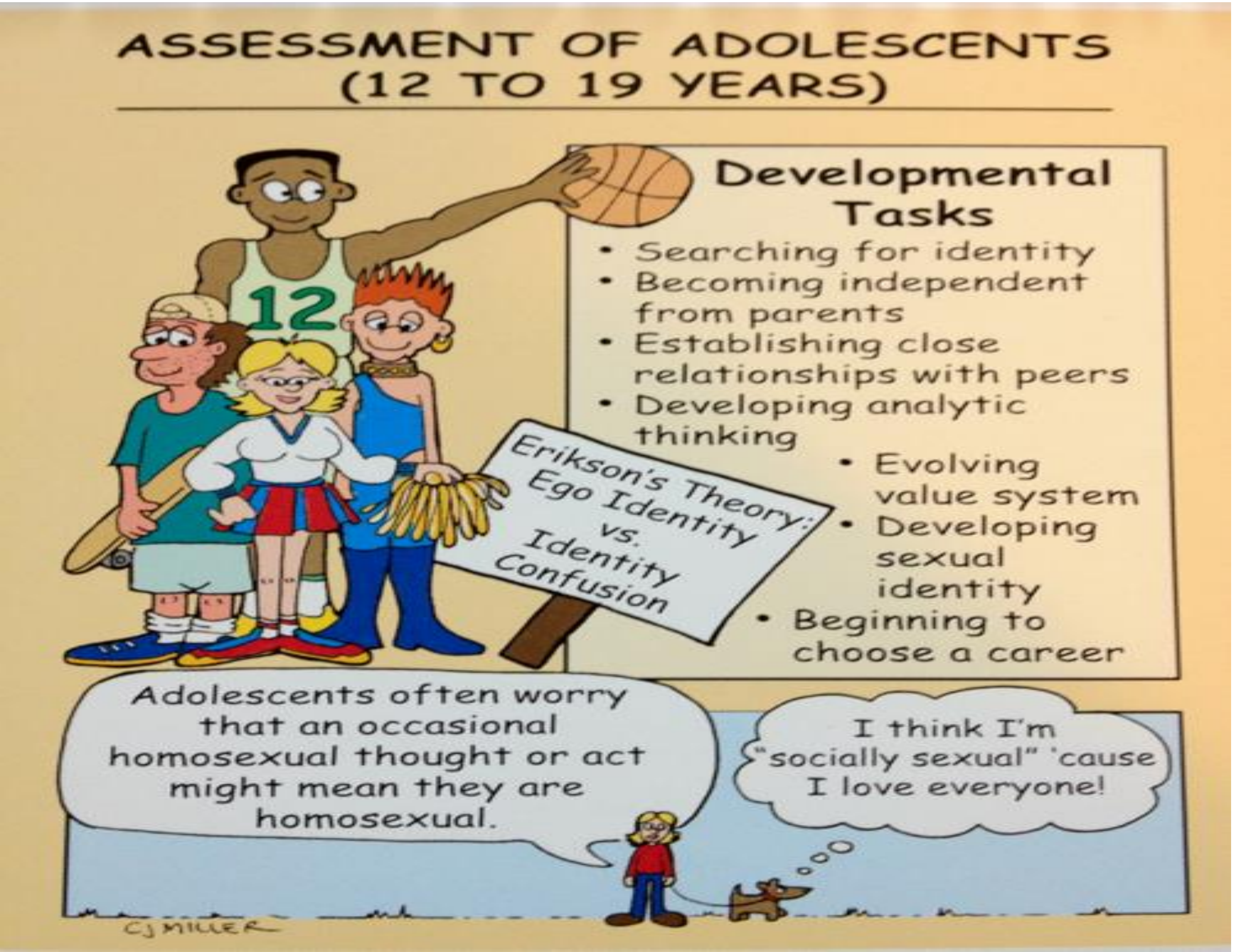
Lyudmyla Kharlamova

CONCLUSION



ASSESSMENT OF ADOLESCENTS (12 TO 19 YEARS)
Developmental Tasks
• Searching for identity
• Becoming independent from parents
• Establishing close relationships with peers
• Developing analytic thinking
• Evolving value system
• Developing sexual identity
• Beginning to choose a career
Erikson's Theory: Ego Identity vs. Identity Confusion
12
Adolescents often worry that an occasional homosexual thought or act might mean they are homosexual.
I think I'm "socially sexual" 'cause I love everyone!
CJ MILLER


THANK
YOU

LINEAR ALGEBRA

MATHEMATICS

INNER PRODUCT SPACES

- Inner product represented by angle brackets $\langle \mathbf{u}, \mathbf{v} \rangle$

Let **u**, **v**, and **w** be vectors in a vector space *V*, and let *c* be any scalar. An inner product on *V* is a function that associates a real number $\langle \mathbf{u}, \mathbf{v} \rangle$ with each pair of vectors **u** and **v** and satisfies the following axioms:

(1) $\langle \mathbf{u}, \mathbf{v} \rangle = \langle \mathbf{v}, \mathbf{u} \rangle$

(2) $\langle \mathbf{u}, \mathbf{v} + \mathbf{w} \rangle = \langle \mathbf{u}, \mathbf{v} \rangle + \langle \mathbf{u}, \mathbf{w} \rangle$

(3) $c \langle \mathbf{u}, \mathbf{v} \rangle = \langle c\mathbf{u}, \mathbf{v} \rangle$

(4) $\langle \mathbf{v}, \mathbf{v} \rangle \geq 0$

(5) $\langle \mathbf{v}, \mathbf{v} \rangle = 0$ if and only if $\mathbf{v} = \mathbf{0}$

- Note:

$$\mathbf{u} \cdot \mathbf{v} = \text{dot product (Euclidean inner product for } R^n)$$

$$<\mathbf{u}\,,\mathbf{v}> = \text{general inner product for a vector space } V$$

- Note:

A vector space *V* with an inner product is called an inner product space

$$\text{Vector space: } (V,\ +,\ \cdot)$$

$$\text{Inner product space: } (V,\ +,\ \cdot\ ,\ <,\ >)$$

- Ex : The Euclidean inner product for R^n

  Show that the dot product in R^n satisfies the four axioms of an inner product

Sol:

$$\mathbf{u} = (u_1, u_2, \cdots, u_n) \quad , \quad \mathbf{v} = (v_1, v_2, \cdots, v_n)$$

$$\langle \mathbf{u}, \mathbf{v} \rangle = \mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + \cdots + u_n v_n$$

- Ex : A different inner product for R^n

Show that the following function defines an inner product on R^2. Given $\mathbf{u}=(u_1, u_2)$ and $\mathbf{v}=(v_1, v_2)$,

$$\langle \mathbf{u}, \mathbf{v} \rangle = u_1v_1 + 2u_2v_2$$

Sol:

$$(1)\quad \langle \mathbf{u}, \mathbf{v} \rangle = u_1v_1 + 2u_2v_2 = v_1u_1 + 2v_2u_2 = \langle \mathbf{v}, \mathbf{u} \rangle$$

$$(2)\quad \mathbf{w} = (w_1, w_2)$$

$$\begin{aligned}\Rightarrow \langle \mathbf{u}, \mathbf{v}+\mathbf{w} \rangle &= u_1(v_1+w_1) + 2u_2(v_2+w_2) \\ &= u_1v_1 + u_1w_1 + 2u_2v_2 + 2u_2w_2 \\ &= (u_1v_1 + 2u_2v_2) + (u_1w_1 + 2u_2w_2) \\ &= \langle \mathbf{u}, \mathbf{v} \rangle + \langle \mathbf{u}, \mathbf{w} \rangle\end{aligned}$$

$$(3)\quad c\langle \mathbf{u}, \mathbf{v}\rangle = c(u_1v_1 + 2u_2v_2) = (cu_1)v_1 + 2(cu_2)v_2 = \langle c\mathbf{u}, \mathbf{v}\rangle$$

$$(4)\quad \langle \mathbf{v}, \mathbf{v}\rangle = v_1^{\,2} + 2v_2^{\,2} \geq 0$$

$$(5)\quad \langle \mathbf{v}, \mathbf{v}\rangle = 0 \Rightarrow v_1^{\,2} + 2v_2^{\,2} = 0 \Rightarrow v_1 = v_2 = 0 \quad (\mathbf{v} = \mathbf{0})$$

- Note: Example can be generalized such that

$$\langle \mathbf{u}, \mathbf{v}\rangle = c_1u_1v_1 + c_2u_2v_2 + \cdots + c_nu_nv_n, \text{ for all } c_i > 0$$

can be an inner product on R^n

- Ex : A function that is not an inner product

Show that the following function is not an inner product on R^3

$$\langle \mathbf{u} , \mathbf{v} \rangle = u_1 v_1 - 2u_2 v_2 + u_3 v_3$$

Sol:

Let

$$\mathbf{v} = (1 , 2 , 1)$$

Then $\langle \mathbf{v} , \mathbf{v} \rangle = (1)(1) - 2(2)(2) + (1)(1) = -6 < 0$

Axiom 4 is not satisfied

Thus this function is not an inner product on R^3

- Theorem: Properties of inner products

Let **u**, **v**, and **w** be vectors in an inner product space *V*, and let *c* be any real number

(1)

(2) $$\langle \mathbf{0}, \mathbf{v} \rangle = \langle \mathbf{v}, \mathbf{0} \rangle = 0$$

(3)

$$\langle \mathbf{u} + \mathbf{v}, \mathbf{w} \rangle = \langle \mathbf{u}, \mathbf{w} \rangle + \langle \mathbf{v}, \mathbf{w} \rangle$$

$$\langle \mathbf{u}, c\mathbf{v} \rangle = c \langle \mathbf{u}, \mathbf{v} \rangle$$

※ The definition of norm (or length), distance, angle, orthogonal, and normalizing for general inner product spaces closely parallel to those based on the dot product in Euclidean *n*-space

- Norm (length) of **u**:

$$\|\mathbf{u}\| = \sqrt{\langle \mathbf{u}, \mathbf{u} \rangle}$$

- Distance between **u** and **v**:

$$d(\mathbf{u}, \mathbf{v}) = \|\mathbf{u} - \mathbf{v}\| = \sqrt{\langle \mathbf{u} - \mathbf{v}, \mathbf{u} - \mathbf{v} \rangle}$$

- Angle between two nonzero vectors **u** and **v**:

$$\cos\theta = \frac{\langle \mathbf{u}, \mathbf{v} \rangle}{\|\mathbf{u}\| \, \|\mathbf{v}\|}, \; 0 \le \theta \le \pi$$

- Orthogonal: $(\mathbf{u} \perp \mathbf{v})$

u and **v** are orthogonal if $\langle \mathbf{u}, \mathbf{v} \rangle = 0$

- Normalizing vectors

(1) If $\|\mathbf{v}\| = 1$, then **v** is called a unit vector

(Note that $\|\mathbf{v}\|$ is defined as $\sqrt{\langle \mathbf{v}, \mathbf{v} \rangle}$)

(2)

$$\mathbf{v} \neq \mathbf{0} \xrightarrow{\text{Normalizing}} \frac{\mathbf{v}}{\|\mathbf{v}\|}$$

(the unit vector in the direction of **v**)

(if **v** is not a zero vector)

- Ex : An inner product in the polynomial space

For $p = a_0 + a_1x + \cdots + a_nx^n$ and $q = b_0 + b_1x + \cdots + b_nx^n$,

and $\langle p\, ,\, q\rangle \equiv a_0b_0 + a_1b_1 + \cdots + a_nb_n$ is an inner product

Let $p(x) = 1 - 2x^2,\ q(x) = 4 - 2x + x^2$ be polynomials in P_2

(a) $\langle p\, ,\, q\rangle = ?$ (b) $\|q\| = ?$ (c) $d(p\, ,\, q) = ?$

Sol:

(a) $\langle p\, ,\, q\rangle = (1)(4) + (0)(-2) + (-2)(1) = 2$

(b) $\|q\| = \sqrt{\langle q\, ,\, q\rangle} = \sqrt{4^2 + (-2)^2 + 1^2} = \sqrt{21}$

(c) $\because\ p - q = -3 + 2x - 3x^2$

$\therefore d(p\, ,\, q) = \|p - q\| = \sqrt{\langle p - q, p - q\rangle}$

$= \sqrt{(-3)^2 + 2^2 + (-3)^2} = \sqrt{22}$

- Properties of norm: (the same as the properties for the dot product in R^n)

(1) $\|\mathbf{u}\| \geq 0$

(2) $\|\mathbf{u}\| = 0$ if and only if $\mathbf{u} = \mathbf{0}$

(3) $\|c\mathbf{u}\| = |c|\,\|\mathbf{u}\|$

- Properties of distance: (the same as the properties for the dot product in R^n)

(1) $d(\mathbf{u}, \mathbf{v}) \geq 0$

(2) $d(\mathbf{u}, \mathbf{v}) = 0$ if and only if $\mathbf{u} = \mathbf{v}$

(3) $d(\mathbf{u}, \mathbf{v}) = d(\mathbf{v}, \mathbf{u})$

- Theorem :

Let **u** and **v** be vectors in an inner product space *V*

(1) Cauchy-Schwarz inequality:

$$|\langle \mathbf{u}, \mathbf{v} \rangle| \le \|\mathbf{u}\| \, \|\mathbf{v}\|$$

(2) Triangle inequality:

$$\|\mathbf{u} + \mathbf{v}\| \le \|\mathbf{u}\| + \|\mathbf{v}\|$$

(3) Pythagorean theorem:

u and **v** are orthogonal if and only if

$$\|\mathbf{u} + \mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2$$

ORTHONORMAL BASES:

- Orthogonal set

  A set S of vectors in an inner product space V is called an orthogonal set if every pair of vectors in the set is orthogonal

$$S = \{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_n\} \subseteq V$$

$$\langle \mathbf{v}_i, \mathbf{v}_j \rangle = 0, \text{ for } i \neq j$$

- Orthonormal set

  An orthogonal set in which each vector is a unit vector is called orthonormal set

$$S = \{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_n\} \subseteq V$$

$$\begin{cases} \text{For } i = j, \ \langle \mathbf{v}_i, \mathbf{v}_j \rangle = \langle \mathbf{v}_i, \mathbf{v}_i \rangle = \|\mathbf{v}_i\|^2 = 1 \\ \text{For } i \neq j, \ \langle \mathbf{v}_i, \mathbf{v}_j \rangle = 0 \end{cases}$$

- Note:
  - If S is also a basis, then it is called an orthogonal basis or an orthonormal basis
  - The standard basis for R^n is orthonormal. For example,

$$S = \{(1,0,0),(0,1,0),(0,0,1)\}$$

is an orthonormal basis for R^3

- Ex : A nonstandard orthonormal basis for R^3

Show that the following set is an orthonormal basis

$$S = \left\{ \underset{\mathbf{v}_1}{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right)}, \quad \underset{\mathbf{v}_2}{\left(-\frac{\sqrt{2}}{6}, \frac{\sqrt{2}}{6}, \frac{2\sqrt{2}}{3} \right)}, \quad \underset{\mathbf{v}_3}{\left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3} \right)} \right\}$$

Sol:

First, show that the three vectors are mutually orthogonal

$$\mathbf{v}_1 \cdot \mathbf{v}_2 = -\tfrac{1}{6} + \tfrac{1}{6} + 0 = 0$$

$$\mathbf{v}_1 \cdot \mathbf{v}_3 = \frac{2}{3\sqrt{2}} - \frac{2}{3\sqrt{2}} + 0 = 0$$

$$\mathbf{v}_2 \cdot \mathbf{v}_3 = -\frac{\sqrt{2}}{9} - \frac{\sqrt{2}}{9} + \frac{2\sqrt{2}}{9} = 0$$

Second, show that each vector is of length 1

$$\|\mathbf{v}_1\| = \sqrt{\mathbf{v}_1 \cdot \mathbf{v}_1} = \sqrt{\tfrac{1}{2} + \tfrac{1}{2} + 0} = 1$$

$$\|\mathbf{v}_2\| = \sqrt{\mathbf{v}_2 \cdot \mathbf{v}_2} = \sqrt{\tfrac{2}{36} + \tfrac{2}{36} + \tfrac{8}{9}} = 1$$

$$\|\mathbf{v}_3\| = \sqrt{\mathbf{v}_3 \cdot \mathbf{v}_3} = \sqrt{\tfrac{4}{9} + \tfrac{4}{9} + \tfrac{1}{9}} = 1$$

Thus S is an orthonormal set

Because these three vectors are linearly independent (you can check by solving $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 = \mathbf{0}$) in R^3 (of dimension 3), by (given a vector space with dimension n, then n linearly independent vectors can form a basis for this vector space), these three linearly independent vectors form a basis for R^3.

$\Rightarrow$ S is a (nonstandard) orthonormal basis for R^3

- Ex : An orthonormal basis for $P_2(x)$

In $P_2(x)$, with the inner product $\langle p, q\rangle = a_0b_0 + a_1b_1 + a_2b_2$,

the standard basis $B = \{1, x, x^2\}$ is orthonormal

Sol:

$$\mathbf{v}_1 = 1 + 0x + 0x^2, \quad \mathbf{v}_2 = 0 + x + 0x^2, \quad \mathbf{v}_3 = 0 + 0x + x^2,$$

Then

$$\langle \mathbf{v}_1, \mathbf{v}_2\rangle = (1)(0) + (0)(1) + (0)(0) = 0$$

$$\langle \mathbf{v}_1, \mathbf{v}_3\rangle = (1)(0) + (0)(0) + (0)(1) = 0$$

$$\langle \mathbf{v}_2, \mathbf{v}_3\rangle = (0)(0) + (1)(0) + (0)(1) = 0$$

$$\|\mathbf{v}_1\| = \sqrt{\langle \mathbf{v}_1, \mathbf{v}_1\rangle} = \sqrt{(1)(1) + (0)(0) + (0)(0)} = 1$$

$$\|\mathbf{v}_2\| = \sqrt{\langle \mathbf{v}_2, \mathbf{v}_2\rangle} = \sqrt{(0)(0) + (1)(1) + (0)(0)} = 1$$

$$\|\mathbf{v}_3\| = \sqrt{\langle \mathbf{v}_3, \mathbf{v}_3\rangle} = \sqrt{(0)(0) + (0)(0) + (1)(1)} = 1$$

- Theorem: Orthogonal sets are linearly independent

  If $S=\{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_n\}$ is an orthogonal set of nonzero vectors in an inner product space *V*, then *S* is linearly independent

Pf:

S is an orthogonal set of nonzero vectors,

$$\text{i.e., } \langle \mathbf{v}_i, \mathbf{v}_j \rangle = 0 \text{ for } i \neq j, \text{ and } \langle \mathbf{v}_i, \mathbf{v}_i \rangle > 0$$

$$\text{For } c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_n\mathbf{v}_n = \mathbf{0}$$

$$\Rightarrow \langle c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_n\mathbf{v}_n, \mathbf{v}_i \rangle = \langle \mathbf{0}, \mathbf{v}_i \rangle = 0 \quad \forall i$$

$$\Rightarrow c_1\langle \mathbf{v}_1, \mathbf{v}_i \rangle + c_2\langle \mathbf{v}_2, \mathbf{v}_i \rangle + \cdots + c_i\langle \mathbf{v}_i, \mathbf{v}_i \rangle + \cdots + c_n\langle \mathbf{v}_n, \mathbf{v}_i \rangle$$

$$= c_i\langle \mathbf{v}_i, \mathbf{v}_i \rangle = 0$$

$$\because \langle \mathbf{v}_i, \mathbf{v}_i \rangle \neq 0 \quad \Rightarrow c_i = 0 \quad \forall i \qquad \therefore S \text{ is linearly independent}$$

- Corollary :

  If V is an inner product space with dimension n, then any orthogonal set of n nonzero vectors is a basis for V

1. if $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ is an orthogonal set of n vectors, then S is linearly independent
2. if $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ is a linearly independent set of n vectors in V (with dimension n), then S is a basis for V

- Ex : Using orthogonality to test for a basis

Show that the following set is a basis for R^4

$$S=\{\overset{\mathbf{v}_1}{(2,3,2,-2)},\overset{\mathbf{v}_2}{(1,0,0,1)},\overset{\mathbf{v}_3}{(-1,0,2,1)},\overset{\mathbf{v}_4}{(-1,2,-1,1)}\}$$

Sol:

$\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3,\mathbf{v}_4$: nonzero vectors

$$\mathbf{v}_1\cdot\mathbf{v}_2=2+0+0-2=0 \qquad \mathbf{v}_2\cdot\mathbf{v}_3=-1+0+0+1=0$$

$$\mathbf{v}_1\cdot\mathbf{v}_3=-2+0+4-2=0 \qquad \mathbf{v}_2\cdot\mathbf{v}_4=-1+0+0+1=0$$

$$\mathbf{v}_1\cdot\mathbf{v}_4=-2+6-2-2=0 \qquad \mathbf{v}_3\cdot\mathbf{v}_4=1+0-2+1=0$$

$\Rightarrow S$ is orthogonal

$\Rightarrow S$ is a basis for R^4

Theorem: Coordinates relative to an orthonormal basis

If $B=\{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_n\}$ is an orthonormal basis for an inner product space *V*, then the unique coordinate representation of a vector w with respect to *B* is

$$\mathbf{w}=\langle\mathbf{w}, \mathbf{v}_1\rangle\mathbf{v}_1+\langle\mathbf{w}, \mathbf{v}_2\rangle\mathbf{v}_2+\cdots+\langle\mathbf{w}, \mathbf{v}_n\rangle\mathbf{v}_n$$

Pf:

$B=\{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_n\}$ is an orthonormal basis for *V*

$$\mathbf{w}=k_1\mathbf{v}_1+k_2\mathbf{v}_2+\cdots+k_n\mathbf{v}_n \in V$$

$$\text{Since } \langle\mathbf{v}_i, \mathbf{v}_j\rangle=\begin{cases}1 & i=j\\ 0 & i\neq j\end{cases}, \text{ then}$$

$$\langle \mathbf{w}, \mathbf{v}_i \rangle = \langle (k_1\mathbf{v}_1 + k_2\mathbf{v}_2 + \cdots + k_n\mathbf{v}_n), \mathbf{v}_i \rangle$$

$$= k_1 \langle \mathbf{v}_1, \mathbf{v}_i \rangle + \cdots + k_i \langle \mathbf{v}_i, \mathbf{v}_i \rangle + \cdots + k_n \langle \mathbf{v}_n, \mathbf{v}_i \rangle$$

$$= k_i \qquad \text{for } i = 1 \text{ to } n$$

$$\Rightarrow \mathbf{w} = \langle \mathbf{w}, \mathbf{v}_1 \rangle \mathbf{v}_1 + \langle \mathbf{w}, \mathbf{v}_2 \rangle \mathbf{v}_2 + \cdots + \langle \mathbf{w}, \mathbf{v}_n \rangle \mathbf{v}_n$$

- Note:

If $B = \{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_n\}$ is an orthonormal basis for *V* and $\mathbf{w} \in V$,

Then the corresponding coordinate matrix of **w** relative to *B* is

$$[\mathbf{w}]_B = \begin{bmatrix} \langle \mathbf{w}, \mathbf{v}_1 \rangle \\ \langle \mathbf{w}, \mathbf{v}_2 \rangle \\ \vdots \\ \langle \mathbf{w}, \mathbf{v}_n \rangle \end{bmatrix}$$

- Ex

  For **w** = (5, –5, 2), find its coordinates relative to the standard basis for R^3

$$\langle \mathbf{w}, \mathbf{v}_1 \rangle = \mathbf{w} \cdot \mathbf{v}_1 = (5, -5, 2) \cdot (1, 0, 0) = 5$$

$$\langle \mathbf{w}, \mathbf{v}_2 \rangle = \mathbf{w} \cdot \mathbf{v}_2 = (5, -5, 2) \cdot (0, 1, 0) = -5$$

$$\langle \mathbf{w}, \mathbf{v}_3 \rangle = \mathbf{w} \cdot \mathbf{v}_3 = (5, -5, 2) \cdot (0, 0, 1) = 2$$

$$\Rightarrow [\mathbf{w}]_B = \begin{bmatrix} 5 \\ -5 \\ 2 \end{bmatrix}$$

- Ex : Representing vectors relative to an orthonormal basis

Find the coordinates of **w** = (5, –5, 2) relative to the following orthonormal basis for R^3

$$B = \{\underset{\mathbf{v}_1}{(\tfrac{3}{5}, \tfrac{4}{5}, 0)}, \underset{\mathbf{v}_2}{(-\tfrac{4}{5}, \tfrac{3}{5}, 0)}, \underset{\mathbf{v}_3}{(0, 0, 1)}\}$$

Sol:

$$\langle \mathbf{w}, \mathbf{v}_1 \rangle = \mathbf{w} \cdot \mathbf{v}_1 = (5, -5, 2) \cdot (\tfrac{3}{5}, \tfrac{4}{5}, 0) = -1$$

$$\langle \mathbf{w}, \mathbf{v}_2 \rangle = \mathbf{w} \cdot \mathbf{v}_2 = (5, -5, 2) \cdot (-\tfrac{4}{5}, \tfrac{3}{5}, 0) = -7$$

$$\langle \mathbf{w}, \mathbf{v}_3 \rangle = \mathbf{w} \cdot \mathbf{v}_3 = (5, -5, 2) \cdot (0, 0, 1) = 2$$

$$\Rightarrow [\mathbf{w}]_B = \begin{bmatrix} -1 \\ -7 \\ 2 \end{bmatrix}$$

- Gram-Schmidt orthonormalization process:

$B=\{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_n\}$ is a basis for an inner product space V

$$\mathbf{u}_1 = \frac{\mathbf{w}_1}{\|\mathbf{w}_1\|} = \frac{\mathbf{v}_1}{\|\mathbf{v}_1\|}$$

$$\mathbf{u}_2 = \frac{\mathbf{w}_2}{\|\mathbf{w}_2\|}, \text{ where } \mathbf{w}_2 = \mathbf{v}_2 - \langle \mathbf{v}_2, \mathbf{u}_1 \rangle \mathbf{u}_1$$

$$\mathbf{u}_3 = \frac{\mathbf{w}_3}{\|\mathbf{w}_3\|}, \text{ where } \mathbf{w}_3 = \mathbf{v}_3 - \langle \mathbf{v}_3, \mathbf{u}_1 \rangle \mathbf{u}_1 - \langle \mathbf{v}_3, \mathbf{u}_2 \rangle \mathbf{u}_2$$

$$\vdots$$

$$\mathbf{u}_n = \frac{\mathbf{w}_n}{\|\mathbf{w}_n\|}, \text{ where } \mathbf{w}_n = \mathbf{v}_n - \sum_{i=1}^{n-1} \langle \mathbf{v}_n, \mathbf{u}_i \rangle \mathbf{u}_i$$

$$\Rightarrow \{\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_n\} \text{ is an orthonormal basis for } V$$

ORTHOGONAL COMPLEMENT

- Orthogonal complement of *V*:

Let *S* be a subspace of an inner product space *V*

(a) A vector **v** in *V* is said to be orthogonal to *S*, if **v** is orthogonal to every vector in *S*, i.e., $\langle \mathbf{v}, \mathbf{w} \rangle = 0, \forall \mathbf{w} \in S$

(b) The set of all vectors in *V* that are orthogonal to *S* is called the orthogonal complement of *S*

$$S^{\perp} = \{\mathbf{v} \in V \mid \langle \mathbf{v}, \mathbf{w} \rangle = 0, \forall \mathbf{w} \in S\}$$

$$(S^{\perp} \text{ (read "}S\text{ perp"))}$$

- Notes:

(1) $\left(\{\mathbf{0}\}\right)^{\perp} = V$ (2) $V^{\perp} = \{\mathbf{0}\}$

(This is because $\langle \mathbf{0}, \mathbf{v} \rangle = 0$ for any vector **v** in *V*)

- **Notes:**

  Given S to be a subspace of V,

  (1) $S^{\perp}$ is a subspace of V

  (2) $S \cap S^{\perp} = \{\mathbf{0}\}$

  (3) $(S^{\perp})^{\perp} = S$

- Ex:

  If $V = R^2, \quad S = x\text{-axis}$

  Then (1) $S^{\perp} = y$-axis is a subspace of R^2

  (2) $S \cap S^{\perp} = \{(0,0)\}$

  (3) $(S^{\perp})^{\perp} = S$

- Direct sum

  Let S_1 and S_2 be two subspaces of V. If each vector $\mathbf{x} \in V$ can be uniquely written as a sum of a vector $\mathbf{v}_1$ from S_1 and a vector $\mathbf{v}_2$ from S_2, i.e., $\mathbf{x} = \mathbf{v}_1 + \mathbf{v}_2$, then V is the **direct sum** of S_1 and S_2, and we can write

$$V = S_1 \oplus S_2$$

- Theorem : Properties of orthogonal subspaces

Let S be a subspace of V (with dimension n). Then the following properties are true

(1) $\dim(S)+\dim(S^{\perp})=n$

(2) $V=S\oplus S^{\perp}$

(3) $(S^{\perp})^{\perp}=S$

ENGLISH

BOOK REVIEW

A **book review** is a form of literary criticism in which a book is analyzed based on content, style, and merit . A book review may be a primary source, opinion piece, summary review or scholarly review. Books can be reviewed for printed periodicals, magazines and newspapers, as school work, or for book web sites on the Internet. A book review's length may vary from a single paragraph to a substantial essay. Such a review may evaluate the book on the basis of personal taste.

An analytic or critical review of a book or article is not primarily a summary; rather, it **comments on** and **evaluates** the work in the light of specific issues and theoretical concerns in a course.The literature review puts together a set of such commentaries to map out the current range of positions on a topic; then the writer can define his or her own position in the rest of the paper. Keep questions like these in mind as you read, make notes, and write the review

1. What is the specific topic of the book or article? What overall purpose does it seem to have? For what readership is it written? (The preface, acknowledgements, bibliography and index can be helpful in answering these questions. Don't overlook facts about the author's background and the circumstances of the book's creation and publication.)

2. Does the author state an explicit thesis? Does he or she noticeably have an axe to grind? What are the theoretical assumptions? Are they discussed explicitly? (Again, look for statements in the preface, etc. and follow them up in the rest of the work.)

3. What exactly does the work contribute to the overall topic of your course? What general problems and concepts in your discipline and course does it engage with?

4. What kinds of material does the work present (e.g. primary documents or secondary material, literary analysis, personal observation, quantitative data, biographical or historical accounts)?

5. How is this material used to demonstrate and argue the thesis? (As well as indicating the overall structure of the work, your review could quote or summarize specific passages to show the characteristics of the author's presentation, including writing style and tone.)

6. Are there alternative ways of arguing from the same material? Does the author show awareness of them? In what respects does the author agree or disagree?

7. What theoretical issues and topics for further discussion does the work raise?

8. What are your own reactions and considered opinions regarding the work?